Relation between the $2\nu\beta\beta$ and $0\nu\beta\beta$ nuclear matrix elements

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Abstract. A formal relation between the GT part of the nuclear matrix elements $M^{0\nu}_{GT}$ of $0\nu\beta\beta$ decay and the closure matrix elements $M^{2\nu}_{cl}$ of $2\nu\beta\beta$ decay is established. This relation is based on the integral representation of these quantities in terms of their dependence on the distance r between the two nucleons undergoing transformation. We also discuss the difficulties in determining the correct values of the closure $2\nu\beta\beta$ decay matrix elements.

Keywords: Double beta decay, nuclear matrix elements
PACS: 21.60.-n,23.40.Hc

INTRODUCTION

The $2\nu\beta\beta$ decay mode of most candidate nuclei has been observed and its half-life determined. Thus, the corresponding nuclear matrix elements $M^{2\nu}$ are known. These matrix elements, of dimension energy^{-1}, vary abruptly between nuclei with different Z and A; they exhibit pronounced shell effects. In contrast, the fundamentally more important $0\nu\beta\beta$ decay mode have not been reliably observed so far. Therefore, the corresponding dimensionless nuclear matrix elements $M^{0\nu}$ must be evaluated theoretically. These calculated quantities, whether based on the QRPA [1, 2, 3, 4], nuclear shell model [5, 6, 7], or the Interacting Boson Model [8], do not show such a variability; instead they vary relatively smoothly between nuclei with different Z, and A. Evaluation of the $M^{0\nu}$ using the Generator Coordinate Method [9] or the Projected Hartree-Fock-Bogolyubov Method [10] vary smoothly with Z and A as well.

However, the calculated values of the matrix elements $M^{0\nu}$ using different approximations do not agree with each other perfectly, differences of a factor of about two exist. Moreover, given the fundamental importance of these quantities for the planning and interpreting the $0\nu\beta\beta$ decay experiments, it would be good to have independent observables that could be linked to their magnitude. In that context we wish to address in this work several questions:

- Can one understand intuitively the different behavior of $M^{0\nu}$ and $M^{2\nu}$ when of Z and A are varied ?
- Is there a formal relation between $M^{0\nu}$ and $M^{2\nu}$?
- If such relation exists can it be used to test the calculated values of the $0\nu\beta\beta$ matrix elements ?

Workshop on Calculation of Double-Beta-Decay Matrix Elements
AIP Conf. Proc. 1417, 139-143 (2011); doi: 10.1063/1.3671053
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In discussing these problems we basically follow the work published by us and our collaborators earlier in Ref. [11].

FORMALISM

Assuming that the $0\nu\beta\beta$ decay is caused by exchange of the light Majorana neutrinos, the nuclear matrix element consists of Gamow-Teller, Fermi and Tensor parts,

$$M^{0\nu} = M^{0\nu}_{GT} - \frac{M^{0\nu}_F}{g_A^2} + M^{0\nu}_T \equiv M^{0\nu}_{GT}(1 + \chi_F + \chi_T) \ , \tag{1}$$

where χ_F and χ_T are matrix element ratios that are smaller than unity and, presumably, less dependent on the details of the applied nuclear model.

In the following we concentrate on the GT part, $M^{0\nu}_{GT}$, which can be somewhat symbolically written as

$$M^{0\nu}_{GT} = \langle f|\Sigma_{lk}\vec{\sigma}_l \cdot \vec{\sigma}_k \tau_l^+ \tau_k^+ H(r_{lk},\bar{E})|i\rangle \ , \tag{2}$$

where $H(r_{lk},\bar{E})$ is the neutrino potential and r_{lk} is the relative distance between the two neutrons that are transformed in the decay into the two protons.

The dependence of $M^{0\nu}$ on the distance r_{lk} is described by the function $C^{0\nu}(r)$ (first introduced in [12] see also [3, 5])

$$C^{0\nu}_{GT}(r) = \langle f|\Sigma_{lk}\vec{\sigma}_l \cdot \vec{\sigma}_k \tau_l^+ \tau_k^+ \delta(r - r_{lk}) H(r_{lk},\bar{E})|i\rangle \ , \quad M^{0\nu}_{GT} = \int_0^\infty C^{0\nu}_{GT}(r) dr \ , \tag{3}$$

where $\delta(x)$ is the Dirac delta function.

In analogy to Eq. (3) we can define for the case of the $2\nu\beta\beta$ decay a new function

$$C^{2\nu}_{cl}(r) = \langle f|\Sigma_{lk}\vec{\sigma}_l \cdot \vec{\sigma}_k \delta(r - r_{lk}) \tau_l^+ \tau_k^+|i\rangle \ , \quad M^{2\nu}_{cl} = \int_0^\infty C^{2\nu}_{cl}(r) dr \ . \tag{4}$$

This function, therefore, is related to the dimensionless *closure* matrix element for the $2\nu\beta\beta$ decay, not the true, dimension energy^{-1} matrix element $M^{2\nu}$ that contains the corresponding energy denominators. While the matrix elements $M^{2\nu}$ and $M^{2\nu}_{cl}$ get contributions only from the 1^+ intermediate states, the function $C^{2\nu}_{cl}(r)$ gets contributions from all intermediate multipoles. This is the consequence of the δ function in the definition of $C^{2\nu}_{cl}(r)$. Naturally, when integrated over r only the contributions from the 1^+ are nonvanishing.

It is now clear that, by construction,

$$C^{0\nu}_{GT}(r) = H(r,\bar{E}) \times C^{2\nu}_{cl}(r) \ , \tag{5}$$

which is valid for any shape of the neutrino potential $H(r,\bar{E})$. Thus, if $C^{2\nu}_{cl}(r)$ is known, $C^{0\nu}_{GT}(r)$ and therefore also $M^{0\nu}_{GT}$ can be easily determined since the neutrino potential $H(r,\bar{E})$ is known and only weakly dependent on the average excitation energy $\bar{E}$. The equation (5) represents the basic relation between the 0ν and 2ν $\beta\beta$-decay modes.

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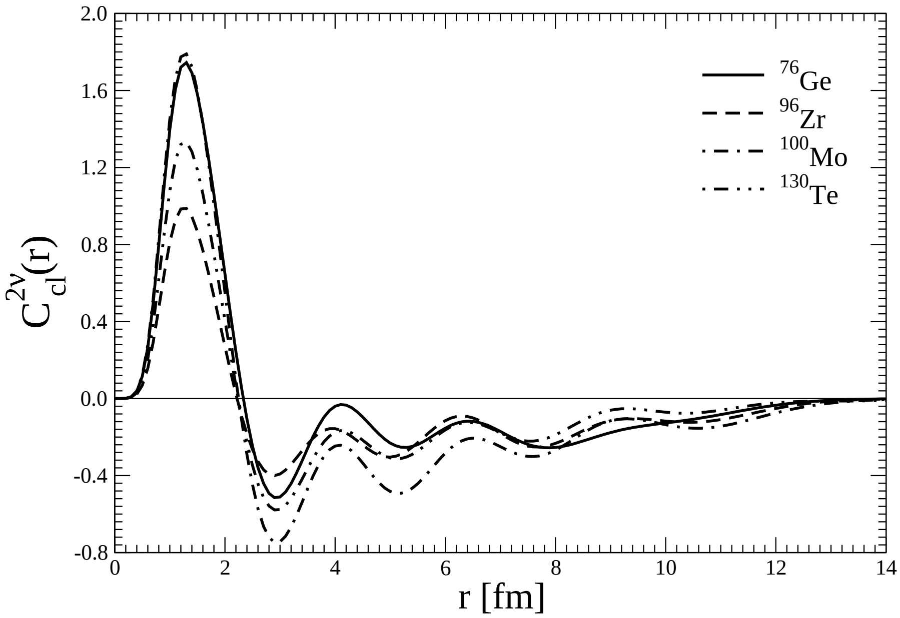

FIGURE 1. The function $C^{2\nu}_{cl}(r)$ for different selected candidate nuclei.

Examples of the function $C^{2\nu}_{cl}(r)$ are shown in Fig. 1 . Note that while the function $C^{2\nu}_{cl}(r)$ has a substantial negative tail past $r \sim 2-3$ fm, these distances contribute very little to $C^{0\nu}_{GT}(r)$ and, therefore also to $M^{0\nu}$. This is a consequence of the shape of the neutrino potential $H(r,\bar{E})$ that decreases fast with increasing values of the distance r.

RESULTS AND DISCUSSION

Remembering that in a nucleus the average distance between nucleons is $\sim$1.2 fm we can somewhat schematically separate the range of the variable r in the functions $C^{0\nu}_{GT}(r)$ and $C^{2\nu}_{cl}(r)$ into the region $r \leq$ 2-3 fm governed by the nucleon-nucleon correlations, while the region $r \geq$ 2-3 fm is governed by nuclear many-body physics. From the form of $C^{0\nu}_{GT}(r)$ it follows that the matrix elements $M^{0\nu}_{GT}$ are almost independent of the "nuclear" region of r and hence one does not expect rapid variations of their value when A or Z of the nucleus is changed. On the other hand, the 2ν closure matrix elements $M^{2\nu}_{cl}$ depend sensitively on that region of r since there is a substantial cancellation between the positive part at $r \leq 2-3$ fm and the negative tail at $r \geq 2-3$ fm. Hence one expects sizable shell effects, i.e. a significant variation of $M^{2\nu}$ and $M^{2\nu}_{cl}$ with A and Z, in agreement with observations. We have thus answered the first two questions posed in the Introduction.

But answering the third question is not so simple. While the nuclear matrix elements $M^{2\nu}$ are simply related to the 2ν half-life $T^{2\nu}_{1/2}$, and are thus known for the nuclei in which $T^{2\nu}_{1/2}$ has been measured, the closure matrix elements $M^{2\nu}_{cl}$ need be determined separately. One can rely on a nuclear model (e.g. QRPA or nuclear shell model), adjust parameters

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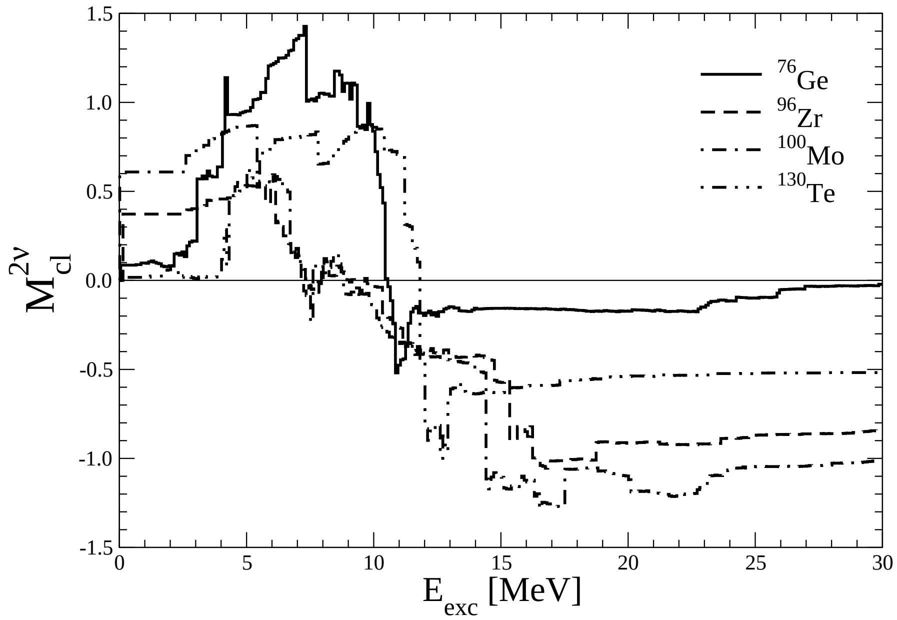

FIGURE 2. Running sums of $M^{2\nu}_{cl}$ for different selected nuclei as a function of the excitation energy of the 1^+ states in the intermediate odd-odd nucleus.

in such a way that the experimental value of $M^{2\nu}$ is correctly reproduced, and use the model to evaluate $M^{2\nu}_{cl}$. Alternatively, one could use the measured β^- and β^+ strength functions and assume coherence (i.e. same signs) among states with noticeable strengths in both channels. In this way an upper limit of $M^{2\nu}_{cl}$ can be obtained. Obviously, neither of these procedures is exact. In the following we leave temporarily aside the important question to which extent the function $C^{2\nu}_{cl}(r)$ is strongly constrained by its integral value $M^{2\nu}_{cl}$.

We have shown in Ref. [11] that, within QRPA, the quantities $M^{2\nu}_{cl}$ are negative in many cases, contrary to simple expectations (because one obviously expects that the average excitation energy is positive). On the other hand, the second method, mentioned above, leads to the positive $M^{2\nu}_{cl}$ by definition. Thus, the two methods disagree with each other.

Since there is a substantial experimental activity devoted to the determination of the $\beta^{\pm}$ strengths [13], it is worthwhile to examine in more detail the somewhat unexpected finding that in many cases $M^{2\nu}$ and $M^{2\nu}_{cl}$ have opposite signs. Obviously, this has to do with the different weights of the individual 1^+ intermediate states in $M^{2\nu}$ and $M^{2\nu}_{cl}$. Because of this the higher energy excited 1^+ states contribute substantially more to $M^{2\nu}_{cl}$ than to $M^{2\nu}$. In Fig. 2 we plot the running sums of $M^{2\nu}_{cl}$ as the function of excitation energy in the intermediate nucleus. One can see that there are negative contributions to the $M^{2\nu}_{cl}$ values that arise from excitation energies $E_{ex} \geq 10$ MeV where it is difficult to explore experimentally the corresponding β^+ strength. Note, moreover, that this is the region of the giant GT resonance, with substantial β^- strength. Hence even a relatively small β^+ strength can have a sizable effect.

The negative contribution to $M^{2\nu}$ and $M^{2\nu}_{cl}$ from relatively high lying 1^+ states seems

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to be a generic effect present in essentially all nuclei. (Note that analogous effect is present in the shell model evaluation of $M^{2\nu}$ for ^{48}Ca, see Ref. [14], Fig. 1.). Thus, it appears that in many cases when the matrix elements $M^{2\nu}$ (with energy denominators, see [11]) are evaluated as a function of the intermediate state excitation energy, the final correct value is reached twice; once at relatively low excitation energy and the second time asymptotically.

These negative contributions to $M^{2\nu}$ and $M^{2\nu}_{cl}$, coming from the vicinity of the giant GT state, clearly exist only in calculations that are able to describe the giant GT resonance. In the restricted shell model calculations (without the full set of the spin-orbit partners) such high lying 1^+ will be absent. Thus, we need to make sure that a noticeable β^+ strength connecting the final nucleus with the 1^+ states in the vicinity of the GT resonance really exists and is not an artifact of our QRPA evaluation. Unless and until this dilemma is resolved it is premature to proceed further in our original program of finding connection between the nuclear matrix elements of the 0ν and 2ν $\beta\beta$-decay modes.

Nevertheless, we have established a formal relation between the nuclear matrix elements $M^{0\nu}_{GT}$ of the neutrinoless $\beta\beta$ decay and the $M^{2\nu}_{cl}$, the closure matrix element for the $2\nu\beta\beta$ decay. We also pose a challenge to both experimentalists studying the charge exchange reactions of the (n,p) type, and to theorists using methods alternative to QRPA to establish whether a noticeable β^+ strength at energies near the giant GT resonance exists or not.

ACKNOWLEDGMENTS

The authors would like to thank the organizers of MEDEX'11 for an opportunity to present their result. The work of P.V. was partially supported by the US Department of Energy under Contract No. DE-FG02-88ER40397. F.Š acknowledges the support by the VEGA Grant agency under the contract No. 1/0249/03.

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